Uniwersytet Wrocławski

FACULTY OF PHYSICS AND ASTRONOMY

INSTITUTE OF THEORETICAL PHYSICS
pl. Maksa Borna 9
50-204 Wroclaw | Poland

Tel. +48 71 375 94 08 | +48 71 375 92 86 | + 48 71 375 95 66
Fax +48 71 321 44 54

www.ift.uni.wroc.pl

Review

on the dissertation of M.Sc.Eng. Agata Marta Zdanowicz
"Collective properties of superfluid fermionic mixture in neutron star crust",
submitted for the degree of Doctor of Philosophy (PhD) in the discipline of Physical Sciences

The dissertation is devoted to a comprehensive investigation of the collective properties of nuclear matter in the neutron star crust on the basis of time-dependent density functional theory, whereby the candidate has developed and applied the method of numerical simulations.

The dissertation has a clear and logic structure and is formulated in very good English.

After a short introductory chapter where the aims and scope of the Thesis is outlined, the second chapter is devoted to the discussion of the different aspects of strongly interacting nuclear matter in the neutron star interior. Of all different layers that are depicted in Fig. 2.1 (note: reference [14] to Jim Lattimer should be replaced by the original source created by Dany Page, available at https://www.astroscu.unam.mx/neutrones/NS-Picture/NStar/NStar_l.gif), the thesis is devoted to that of the inner crust.

In the remaining four sections of this chapter, mgr Zdanowicz gives an overview on the fundamentals for understanding the physics of the inner crust of neutron stars, for which in particular the basic concepts of superfluidity are important: Bose-Einstein condensation, BCS theory of pairing and superconductivity/superfluidity, formation of quantum vortex lattices and their dependence on the rotation frequency which is essential for understanding the pulsar glitch phenomenon as an observable proof for the presence of nuclear superfluidity in the neutron star crust.

The third chapter is the core of the Thesis, where the theoretical basis is outlined. It begins with the standard of mean-field approximations, the Hartree-Fock (HF) method which for superfluid systems is generalized to the HF-Bogoliubov (HFB) approach, which in the case of spin symmetry can be simplified to the Bogoliubov-deGennes (BdG) theory. For interacting many-body systems in nonequilibrium the solution of these systems of equations becomes a demanding challenge. Therefore, the methods of Density Functional Theory (DFT) have been developed, based on the Hohenberg-Kohn theorem and the Kohn-Sham formalism which are summarized in Section 3.2. In Section 3.3, the DFT is developed for superfluid systems, leading to the time-dependent density functional theory (TDDFT) and time-dependent HFB equations. In order to apply this theory to nuclear systems, the energy functions have to be defined, this is done in Section 3.4. Most prominent in the theory of nuclear matter are zero-range density-dependent potentials of the Skyrme type which are developed in presence of nucleon pairing to the Brussels-Montreal Skyrme (BSk) Functionals as the "gold standard" in nuclear DFT; a summary is provided in Section 3.5. In Section 3.6 mgr Zdanowicz describes the Warsaw-BSk (W-BSk) Toolkit software package which is the basis for obtaining actual results on collective properties of the superfluid neutron star crust.

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Exploiting the W-BSk toolkit, three classes of applications have been studied and the results are discussed in Section 4 (linear movement of an impurity in nuclear matter, published in PRX(2024)), Section 5 (oscillations of an impurity in neutron matter, published in PRC (2025)), and in Section 6 (vortex-nucleus interaction). The results of this last last section are not yet published, but they are very interesting for understanding observed pecularities of pulsars as spinning neutron stars, such as glitches, which depend on the pinning and tension forces that are calculated here.

The unique achievement and precious contribution for the community of neutron star theorists and astrophysicists as reported in this Thesis is the TDDFT based W-BSk toolkit program package for performing numerical simulations of neutron star crust properties. The development of this new software, as demonstrated in this Thesis, represents a significant new achievement in the field of modern neutron star physics. It allows to extract crucial parameters of effective neutron star models (e.g. effective mass parameter) and to examine various dynamical processes involving nuclei and vortices in the superfluid medium of the inner crust of neutron stars.

These new achievements as presented in the PhD Thesis by mgr Agata Zdanowicz are prominently published in two articles in world-leading journals in the field:

1) in Phys. Rev. X (2024), the TDDFT approach underlying the W-BSk toolkit was published and as a numerical experiment the motion of a nuclear impurity accelerated through a neutron superfluid by a constant force has been considered. New insights are reported on the effective mass, the velocity scales and dissipative channels and mechanisms for generating topological defects. A particularly new result was a low-lying excitation of a breathing mode analogous to the GDR accompanying the impurity motion.

2) in Phys. Rev. C (2025), the dynamical scheme for extracting the mass parameters of a nucleus moving in a medium has been further developed so that inertial and quadrupole mass parameters could be extracted.

In addition to the journal publication of the results obtained so far with the W-BSk Toolkit, this numerical tool has been made publicly available on the ZENODO repository: https://doi.org/10.5281/zenodo.15808859

In conclusion, the body of work presented in this dissertation, through its methodological rigor, novel findings, and significant contribution to the field, fully satisfies all the requirements for the Doctor of Philosophy (PhD) degree. Therefore, I recommend the Faculty of Physics of the Warsaw University of Technology to accept this Thesis and to award mgr Agata Zdanowicz with the PhD degree in the discipline of Physical Sciences.

David Blaschke

Wroclaw, 05.05.2026 Prof. dr hab. Dr h.c. David Blaschke